

Waveform Design for Radar-Embedded Communications Based on Weighted-Combining

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Abstract: Radar-embedded communication (REC) is a covert communication method for hiding communication information in radar echo signals. The key in this field is the waveform design. However, the communication waveform constructed by the traditional strategy is not orthogonal. To solve this problem, this paper proposes an Improved Weighted-Combining (IWC) strategy based on the traditional Weighted-Combining (WC) strategy. Performance of the proposed waveform design strategy is presented in terms of the so-called “low probability of intercept (LPI) metric” and symbol error ratio (SER). The improved strategy demonstrates the best LPI performance while improves reliability performance significantly.

1. Introduction

As covert communication becomes more and more important, radar-embedded communication (REC) is applied, which hides communication information in radar echo signals. Compared with other covert communication technology such as physical layer security [1], chaotic communication [2], noise communication [3] etc, the most competitive performance of REC is that the eavesdropper cannot easily judge the received signals as covert signals.

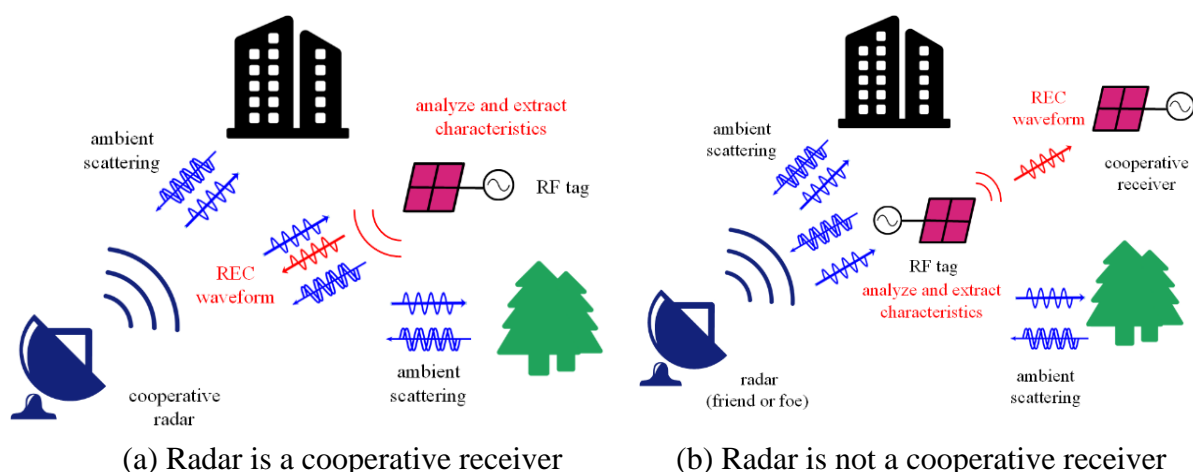


Figure 1. Application scenario of REC

The application scenario of REC is shown in Fig.1. If there is a radar signal (either friend or foe) under the circumstance where the radio frequency (RF) tag/transponder and the cooperative receiver are located, then the RF tag can analyze the radar signal and extract its characteristic information to generate a communication signal with similar characteristics. The tag mixes the communication signal into the radar signal and sends it to the cooperative receiver (either radar or not). The

cooperative receiver possesses prior knowledge of radar signal parameters, communication signal design strategies, and corresponding rules between signals and communication symbols. Therefore, it is viable for the cooperative receiver to extract the communication information through special signal processing methods.

This communication procedure is converted. On one hand, the energy of the communication signal is much lower than that of the radar signal, while the communication signal has similar characteristics to the radar signal. As a result, it is easy to be ignored by the eavesdropper, which means REC possesses the low probability of detection (LPD) performance. On the other hand, it is difficult for the intercept receiver to separate the communication signal and radar echo signal, as well as extract the information, even if the eavesdropper knows the existence of the communication signal. There are two reasons. Firstly, the correlation between the communication signal and the radar signal is high. Secondly, the eavesdropper has no prior information as the cooperative receiver. Therefore, REC also possesses the low probability of intercept (LPI) performance.

Based on the above discussions, the earliest method called inter-pulse REC [4] is to map a communication symbol to multiple radar pulses, and use Doppler phase-shift of multiple radar pulses to make data transmission come true. However, the data rate of this method is very low, only positioning function can be realized. To improve the data rate, Shannon and his team proposed a new method called intra-pulse REC [5] and presented three communication waveform design strategies. In [6], a continuous phase modulation (CPM) based approach is introduced to REC and three different filter designs are proposed. In [7], an alternative mathematical formulation of the maximum signal-to-interference-plus-noise-ratio (SINR) approach is presented, which reduces complexity and addresses computational cost. Characterization of range sidelobe modulation arising from REC is investigated in [8]. A waveform design procedure based on multi-objective optimization is studied in [9] and [10], where a trade-off between reliability and covertness is taken into consideration. Furthermore, part of the research on radar communication integration [11] is applicable to REC.

The key to REC is the waveform design strategy. At present, there are three waveform design strategies, which are eigenvectors-as-waveforms (EAW), weighted-combining (WC) and dominant-projection (DP). Generally, we do not consider the EAW strategy since it is not covert. The WC strategy and the DP strategy are both commonly used in REC. However, the communication waveforms designed by these two strategies are not completely orthogonal, which can affect the performance of communication transmission somehow. This paper observes this problem and proposes an improved waveform design strategy based on WC. The improved strategy demonstrates better reliability and covertness performance than traditional strategies. What's more, the existing work on REC is equally applicable to the improved strategy.

The remainder of this paper is organized as follows. Section 2 introduces the system model. In Section 3, we introduce the traditional WC strategy and propose an improved waveform design strategy. Performance metrics are also presented in this section. Simulation results are given in Section 4 and conclusions are drawn in Section 5.

2. System model

Taking the linear frequency modulated (LFM) radar as an example, the system model can be obtained according to the communication procedure of Fig.1. (Similarly, it applies to other types of radars.) If the channel is an additive white Gaussian noise (AWGN) channel, the signal $r(t)$ received by the receiver can be expressed as

$$r(t) = s(t) * p(t) + \alpha_k c_k(t) * h(t) + n(t) \quad (1)$$

where $s(t)$ is the radar transmit waveform, $p(t)$ is the clutter process of the radar echo, $h(t)$ is the multipath response, $c_k(t)$ is the k th communication waveform transmitted by the tag, $n(t)$ is the environmental noise, α_k is an attenuation constant, $*$ is a convolution process. Here we only consider the case where one communication waveform maps a symbol.

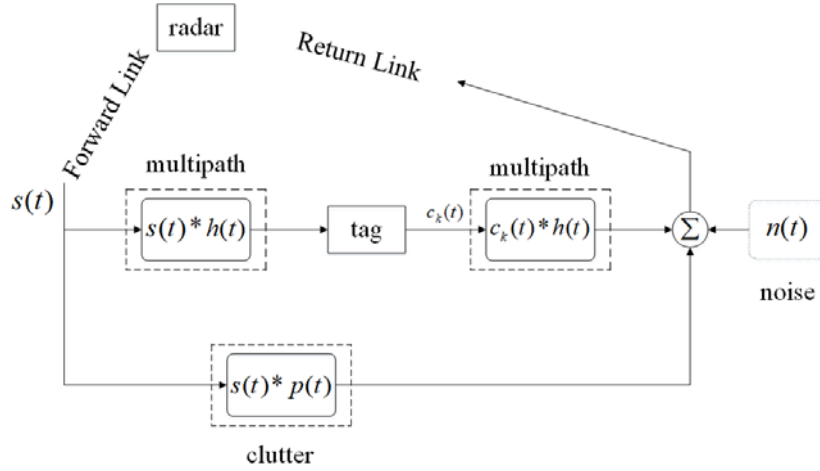


Figure 2. Continuous time signal model

The radar waveform and the communication waveform here are continuous. To facilitate the analysis, they should be sampled and the problem needs to be discussed in the discrete domain. Typically, to realize higher design freedom, the tag and cooperative receiver sample the received signal at a greater sampling rate comparing to the Nyquist sampling rate. Define the number of sampling points as N , the oversampling factor as M_c , then the sampled LFM radar waveform is

$$\mathbf{s} = [s_1, s_2, s_3 \dots s_{NM_c}] \quad (2)$$

where $s_n = e^{j\frac{\pi}{N}(\frac{n}{M_c})^2}$, $n = 1, 2, \dots, NM_c$ (LFM radar), so that the radar echo can be mathematically modeled as the product of the $NM_c \times (2NM_c - 1)$ Toeplitz matrix and the $(2NM_c - 1) \times 1$ clutter column vector (the discrete convolution processing).

$$\mathbf{S} \cdot \mathbf{p} = \begin{bmatrix} s_{NM_c} & s_{NM_c-1} & \cdots & s_1 & \cdots & 0 \\ 0 & s_{NM_c} & \cdots & s_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & s_{NM_c} & \cdots & s_1 \end{bmatrix} \mathbf{p} \quad (3)$$

Here \mathbf{p} is a $(2NM_c - 1) \times 1$ clutter column vector formed by range samples of the surrounding radar scattered echoes. Although we can control the intercept probability by means of utilizing the average power of the ambient radar scattered echo to control the power of the embedded communication signal it is not necessary to know the specific value of \mathbf{p} when designing the communication waveform. So, we only need to analyze the Toeplitz matrix \mathbf{S} .

We perform eigen-decomposition on $\mathbf{S}\mathbf{S}^H$ as

$$\mathbf{S}\mathbf{S}^H = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H \quad (4)$$

where $\mathbf{Q} = [\mathbf{q}_1 \ \mathbf{q}_2 \ \cdots \ \mathbf{q}_{NM_c}]$ is a unitary matrix consisted of the NM_c eigenvectors, $\mathbf{\Lambda}$ is a diagonal matrix composed of the associated eigenvalues (assumed to be in order of increasing magnitude). $(\cdot)^H$ is the Hermit operator, the same below.

We can define the L largest eigenvalues as the dominant region and the remaining eigenvalues as the non-dominant regions. (L is the number of the largest eigenvalues.) The eigenvectors corresponding to the dominant region are more correlated to the radar waveform. Eigen-decomposition can be further expressed as follows:

$$\mathbf{S}\mathbf{S}^H = [\mathbf{Q}_D \quad \mathbf{Q}_{ND}] \begin{bmatrix} \mathbf{\Lambda}_D & 0 \\ 0 & \mathbf{\Lambda}_{ND} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_D^H \\ \mathbf{Q}_{ND}^H \end{bmatrix} \quad (5)$$

where \mathbf{Q}_D represents an $NM_c \times L$ matrix composed of the L eigenvectors corresponding to the dominant region, and \mathbf{Q}_{ND} represents an $NM_c \times (NM_c - L)$ matrix composed of the $NM_c - L$ eigenvectors corresponding to the non-dominant region. Relatively, $\mathbf{\Lambda}_D$ is a $L \times L$ diagonal matrix of the dominant eigenvalues, and $\mathbf{\Lambda}_{ND}$ is an $(NM_c - L) \times (NM_c - L)$ diagonal matrix of non-dominant eigenvalues.

In general, the value of L is determined by the tag and the cooperative receiver. The typical size of the dominant region is $L = N$. Changing the value of the L provides greater freedom in designing communication waveforms and decreases the probability of eavesdropper intercept.

3. Improved Weighted-Combining (IWC) Strategy

In this section, we propose an improved waveform design strategy based on WC to make the communication waveforms orthogonal.

Based on the above system model, the traditional WC strategy [12] can be shown as follows.

Weighting each column vector in \mathbf{Q}_{ND} to generate a communication waveform, such as

$$\mathbf{c}_k = \mathbf{Q}_{ND} \cdot \mathbf{b}_k, k = 1, 2, \dots, K \quad (6)$$

where $\mathbf{Q}_{ND} = [\mathbf{q}_1 \quad \mathbf{q}_2 \quad \dots \quad \mathbf{q}_{NM_c-L}]$ consists of $NM_c - L$ eigenvectors corresponding to the non-dominant region, the same as before. \mathbf{b}_k is a different $(NM_c - L) \times 1$ weight vector known only to the tag and to the cooperative receiver (random column vector). \mathbf{c}_k is the generated communication waveform, which is an $(NM_c - L) \times 1$ vector.

To improve performance, we propose the IWC strategy.

We choose j column vectors satisfying the following conditions randomly in the non-dominant region matrix \mathbf{Q}_{ND} , and then constitute a generator matrix \mathbf{Q}_G . ($\mathbf{Q}_G = [\mathbf{q}_{j1} \quad \mathbf{q}_{j2} \quad \dots \quad \mathbf{q}_{jj}]$, $\mathbf{q}_{j1}, \mathbf{q}_{j2}, \dots, \mathbf{q}_{jj}$ are the selected j column vectors.)

Each column vector can be selected only one time. That is, if \mathbf{q}_i is used to generate the communication waveform \mathbf{c}_1 , it cannot be used to generate other communication waveforms. ($i = 1, 2, \dots, NM_c - L$)

Suppose that we generate m communication waveforms, any \mathbf{Q}_{Gn} ($n = 1, 2, \dots, m$) of $\mathbf{Q}_{G1}, \mathbf{Q}_{G2}, \dots, \mathbf{Q}_{Gm}$ should pick out column vectors corresponding to the larger eigenvalues as many as possible. $\mathbf{Q}_{G1}, \mathbf{Q}_{G2}, \dots, \mathbf{Q}_{Gm}$ are generator matrices. (It ensures that each communication waveform has good performance and the performance among communication waveforms is similar.)

Here, the value of j should not be so small, otherwise it will exhibit a peak in the spectrum, and it is not covert.

The communication waveform \mathbf{c}_k is generated based on the generator matrix \mathbf{Q}_G and the $j \times 1$ random column vector \mathbf{h}_k , as follows

$$\mathbf{c}_k = \mathbf{Q}_{Gk} \cdot \mathbf{h}_k, k = 1, 2, \dots, K \quad (7)$$

For example, if $N = 100$, $M_c = 2$, $L = 100$, then $\mathbf{Q}_{ND} = [\mathbf{q}_1 \quad \mathbf{q}_2 \quad \dots \quad \mathbf{q}_{100}]$. Assuming $K = 4$, the typical IWC waveforms are as follows ($j = 25$):

The generator matrices are:

$$\begin{aligned}
\mathbf{Q}_{G1} &= [\mathbf{q}_1 \ \mathbf{q}_5 \ \mathbf{q}_9 \ \cdots \ \mathbf{q}_{97}] \\
\mathbf{Q}_{G2} &= [\mathbf{q}_2 \ \mathbf{q}_6 \ \mathbf{q}_{10} \ \cdots \ \mathbf{q}_{98}] \\
\mathbf{Q}_{G3} &= [\mathbf{q}_3 \ \mathbf{q}_7 \ \mathbf{q}_{11} \ \cdots \ \mathbf{q}_{99}] \\
\mathbf{Q}_{G4} &= [\mathbf{q}_4 \ \mathbf{q}_8 \ \mathbf{q}_{12} \ \cdots \ \mathbf{q}_{100}]
\end{aligned} \tag{8}$$

The IWC waveforms are:

$$\begin{aligned}
\mathbf{c}_1 &= \mathbf{Q}_{G1} \cdot \mathbf{h}_1 \\
\mathbf{c}_2 &= \mathbf{Q}_{G2} \cdot \mathbf{h}_2 \\
\mathbf{c}_3 &= \mathbf{Q}_{G3} \cdot \mathbf{h}_3 \\
\mathbf{c}_4 &= \mathbf{Q}_{G4} \cdot \mathbf{h}_4
\end{aligned} \tag{9}$$

3.1 Reliability Performance Metric

Define the ratio of the communication signal power to the interference power as the signal-to-interference ratio (SIR), and the ratio of the communication signal power to the environmental noise power as the signal-to-noise ratio (SNR). (At the receiver, we firmly believe that the radar echo is interference.)

Generally, we measure the reliability performance of the designed communication waveform by comparing the symbol error ratio (SER) at the receiver under different SNR and SIR conditions. (Only the waveform decision at the receiver is considered, the symbol mapping of the waveform is ignored.)

3.2 LPI Metric

Considering the worst case, where the eavesdropper knows the REC design principle, radar waveform parameters, oversampling factor M_c and sampling point N .

The eavesdropper first needs to predict the size of the dominant region to generate a corresponding predicted projection matrix \mathbf{P}_{eve} [13]. The predicted projection matrix \mathbf{P}_{eve} is used to process the signal \mathbf{r}_{eve} received by the intercept receiver.

$$\mathbf{z} = \mathbf{P}_{eve} \cdot \mathbf{r}_{eve} \tag{10}$$

where \mathbf{z} is the processed signal, \mathbf{r}_{eve} has an expression similar to (1)

Calculate the normalized correlation $corr$ between the processed signal \mathbf{z} and the actual communication signal \mathbf{c}_k , the larger the $corr$, the easier it is for the eavesdropper to intercept the communication waveform. ($0 \leq corr \leq 1$)

$$corr = \frac{|\mathbf{z}^H \mathbf{c}_k|}{\sqrt{\mathbf{z}^H \mathbf{z}} \sqrt{\mathbf{c}_k^H \mathbf{c}_k}} \tag{11}$$

Although the probability of intercept is not calculated directly, this metric does provide a way to measure the covertness. As a consequence, most papers on REC use it as the LPI metric.

4. Simulation results

In this section, we consider an LFM radar waveform that is over-sampled by oversampling factor $M_c = 2$ and sampling point $N = 100$. The clutter process $p(t)$ and the environmental noise $n(t)$ from (1) are modeled as white Gaussian. The random column vectors for WC, IWC and DP

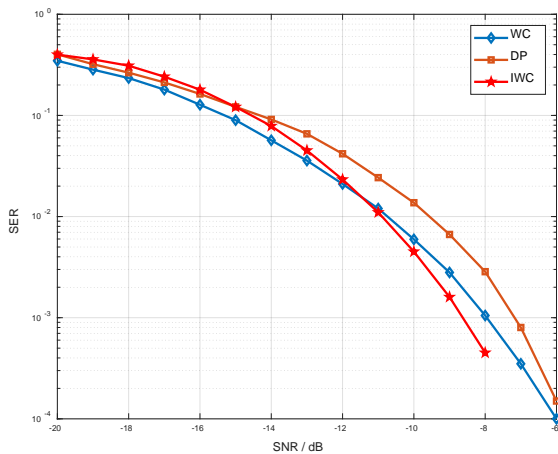
strategies are constant-modulus random-phase and complex Gaussian. The number of communication waveforms is set to $K = 4$ and the size of the dominant region is taken as a typical value ($L = 100$).

In Fig.3, we compare the reliability performance of the IWC strategy and traditional strategies. (We consider the typical IWC strategy mentioned in (8), (9), the same below.) We choose the diagonally loaded decorrelating (DLD) [14] receiver as a sample. The SNR is from -20dB to 10dB, and SIR is respectively set to -25dB and -30dB.

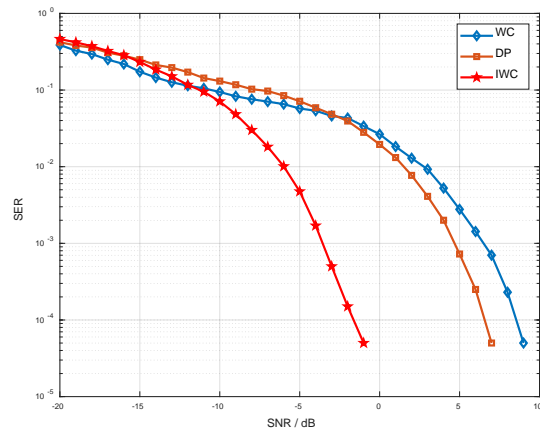
Under the same SIR and SNR condition, the SER performance sequence from good to bad is IWC, WC, DP. As the SIR increases, the SER performance of each waveform is significantly improved. More specifically, in Fig.3(a), IWC outperforms WC with 1.5dB at SER of 10^{-4} . In Fig.3(b), IWC outperforms WC with 10dB at SER of 10^{-4} . Comparing with WC and DP, the SER of IWC is significantly decreased. This is because the IWC waveforms are completely orthogonal.

In Fig.4, we compare the LPI performance of the three strategies. Monte Carlo simulations are performed on more than 1,000,000 trials and then normalized correlations are calculated. The SNR is -5dB, and SIR is respectively set to -25dB and -35dB.

According to normalized correlations, the LPI performance sequence from good to bad is IWC, DP, WC. Regardless of Fig.4(a) or Fig.4(b), IWC basically ensures lower normalized correlations, so IWC has the best LPI performance. What's more, as the SIR increases, the LPI performance of each waveform is improved.

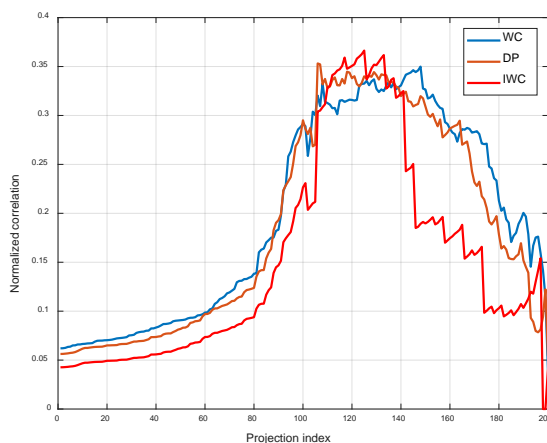


(a) SIR = -25dB

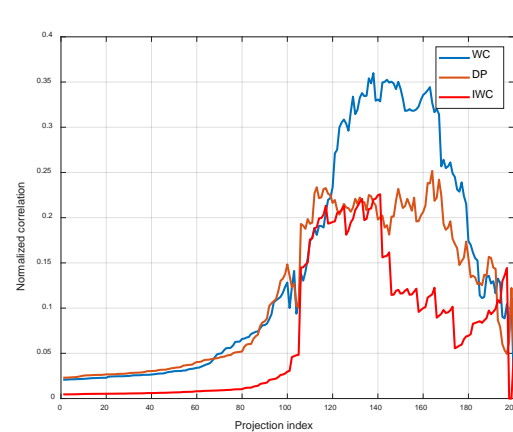


(b) SIR = -30dB

Figure 3. Comparison of reliability performance



(a) SIR = -25dB, SNR = -5dB



(b) SIR = -35dB, SNR = -5dB

Figure 4. Comparison of LPI performance

5. Conclusions

This paper considers the waveform design for REC. To solve the problem that the traditional WC waveform and DP waveform are not orthogonal, this paper proposes an improved waveform design strategy based on WC, which can be called IWC strategy. We compare the reliability performance and LPI performance of IWC waveform and traditional REC waveforms. And IWC waveform demonstrates the best LPI performance while improves reliability performance significantly. Future work in this regard will focus on the duplex design and networking application to achieve engineering application.

Acknowledgments

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